Solve the IVP $y' + \frac{2}{x} y = x^2 y^2 \qquad y(1) = 1$

$y^{-2} y' + \frac{2}{x} y^{-1} = x^2 \qquad \vartheta = y^{-1}$

$\vartheta' = -y^{-2} y'$

$-\vartheta' + \frac{2}{x}\vartheta = x^2 \implies \vartheta' - \frac{2}{x}\vartheta = -x^2 \leftarrow$ lin.

$\mu(x) = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = e^{\ln x^{-2}} = x^{-2}$

$x^{-2}\vartheta' - 2x^{-3}\vartheta = -1 \implies (x^{-2}\vartheta)' = -1$

$x^{-2}\vartheta = \int -1\,dx \implies x^{-2}\vartheta = -x + c$

$\vartheta = x^2(c - x) \implies y^{-1} = x^2(c-x)$

$\implies y = \dfrac{1}{x^2(c-x)} \qquad y(1) = 1$

$1 = \dfrac{1}{c-1} \qquad c - 1 = 1 \implies c = 2$

$y = \dfrac{1}{x^2(2-x)}$

2 $\qquad e^x y^2 \qquad y(0) = 1$

$y^{-2}y' - y^{-1} = e^x \qquad \boxed{\vartheta = y^{-1}} \qquad \vartheta' = -y^{-2}y'$

$-\vartheta' - \vartheta = e^x \qquad \vartheta' + \vartheta = -e^x \leftarrow$ lin.

$\mu(x) = e^{\int 1 dx} = e^x$ Çarp

$e^x\vartheta' + e^x\vartheta = -e^{2x}$

$(e^x\vartheta)' = -e^{2x} \implies e^x\vartheta = -\int e^{2x}dx$

$\implies \vartheta = -\frac{1}{2}e^x + ce^{-x}$

$e^x v = -$

$$y = \frac{1}{v} = \frac{1}{-\frac{1}{2}e^x + ce^{-x}}$$

$$y(0) = 1 \Rightarrow 1 = \frac{1}{-\frac{1}{2} + c} \qquad c - \frac{1}{2} = 1 \Rightarrow c = \frac{3}{2}$$

$$y = \frac{1}{-\frac{1}{2}e^x + \frac{3}{2}e^{-x}}$$

$$y' = \frac{x-y}{x+y} \qquad y(1) = 0$$

$$y' = \frac{x(1-\frac{y}{x})}{x(1+\frac{y}{x})} \Rightarrow y' = \frac{1-\frac{y}{x}}{1+\frac{y}{x}} \longrightarrow \text{hom. eqn}$$

$$\frac{y}{x} = v \Rightarrow y = xv \qquad y' = v + xv'$$

$$v + xv' = \frac{1-v}{1+v} \Rightarrow xv' = \frac{1-v}{1+v} - v = \frac{1-v-v-v^2}{1+v}$$

$$x\frac{dv}{dx} = \frac{1-2v-v^2}{1+v} \Rightarrow \frac{1+v}{1-2v-v^2}dv = \frac{dx}{x}$$

$$\int \frac{1+v}{1-2v-v^2}dv = \int \frac{dx}{x} \Rightarrow \int \frac{-2(1+v)}{1-2v-v^2}dv = -\int 2\frac{dx}{x}$$

$$\Rightarrow \ln|1-2v-v^2| = -2\ln|x| + \ln c_1$$

$$\ln|1-2v-v^2| = \ln(|x|^{-2}c_1)$$

$$1-2v-v^2 = \pm c_1 x^{-2} \qquad 1-2v-v^2 = \frac{c}{x^2}$$

$$1 - \frac{2y}{x} - \frac{y^2}{x^2} = \frac{c}{x^2} \Rightarrow x^2 - 2yx - y^2 = c$$

$$y(1) = 0 \Rightarrow 1 - 0 - 0 = c \Rightarrow c = 1$$

$$\boxed{x^2 - 2xy - y^2 = 1}$$

$$\int \frac{2x}{x^2-1}dx = \ln|x^2-1| + c \qquad \int \frac{f'}{f} = \ln|f|$$

$$\frac{2x}{} = \frac{A}{} + \frac{B}{} = \frac{A(x+1) + B(x-1)}{}$$

$x-1$

$$\frac{2x}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A(x+1) + B(x-1)}{x^2-1}$$

$$\left.\begin{matrix} A+B=2 \\ A-B=0 \end{matrix}\right\} \quad A=B=1$$

$$\int \frac{2x}{x^2-1}dx = \int \frac{1}{x-1} + \frac{1}{x+1}\, dx = \ln|x-1| + \ln|x+1| + c$$

$$= \ln(|x-1|\cdot|x+1|) + c$$

$$= \ln|x^2-1| + c$$

$y' = \dfrac{2x+y+1}{2x+y-2}$ Solve. $\qquad y' = G(ax+by)$

$$G(t) = \frac{t+1}{t-2}$$

$u = 2x+y \qquad u' = 2+y'$

$$\Rightarrow y' = u'-2$$

$$u'-2 = \frac{u+1}{u-2} \qquad u' = \frac{u+1}{u-2} + 2 = \frac{u+1+2u-4}{u-2}$$

$$u' = \frac{3u-3}{u-2} \Rightarrow \frac{u-2}{3u-3}du = dx$$

$$\int \frac{u-2}{3u-3}du = \int dx$$

$$\begin{array}{r|l} u-2 & 3u-3 \\ -\;u-1 & \frac{1}{3} \\ \hline -1 & \end{array}$$

$$\int \frac{1}{3} - \frac{}{3u-3}\, du = \int dx$$

$$\frac{1}{3}u - \frac{1}{3}\ln|3u-3| = x + c$$

$$\int \frac{1}{ax+b} = \frac{1}{a}\ln|ax+b|$$

$$\frac{1}{3}(2x+y) - \frac{1}{3}\ln|6x+3y-3| = x + c$$

Find an integrating of the form $\mu(x,y) = x^m y^n$ and then solve $(5x^2y + 2y^2)dx + (2x^3 + 2xy)dy = 0$

Multiply the equation by $x^m y^n$ to get the exact equation

$$(\underbrace{5x^{m+2}y^{n+1} + 2x^m y^{n+2}}_{M})dx + (\underbrace{2x^{m+3}y^n + 2x^{m+1}y^{n+1}}_{N})dy = 0$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}$$

$$5(n+1)x^{m+2}y^n + 2(n+2)x^m y^{n+1} = 2(m+3)x^{m+2}y^n + 2(m+1)x^m y^{n+1}$$

$5(n+1) = 2(m+3)$

$\cancel{2}(n+2) = \cancel{2}(m+1) \Rightarrow m = n+1$

$5(n+1) = 2(n+4) \Rightarrow 5n+5 = 2n+8 \Rightarrow n=1, \; m=2$

Put $m=2$, $n=1$ in the last eqn

$$(5x^4y^2 + 2x^2y^3)\cdot dx + (2x^5y + 2x^3y^2)\,dy = 0$$

$$\Rightarrow d(x^5y^2) + d\left(\frac{2}{3}x^3y^3\right) = 0 \qquad dF(x,y) = M(x,y)dx + N(x,y)dy = 0$$

$$d(x^5y^2) = 5x^4y^2dx + 2x^5y\,dy \qquad F(x,y) = c$$

$$d\left(x^5y^2 + \frac{2}{3}x^3y^3\right) = 0 \qquad d(f(x)) = f'(x)dx$$

$$x^5y^2 + \frac{2}{3}x^3y^3 = C$$

$$(2xy^2 + \underbrace{4x^3}_{d(x^4)})dx + (2x^2y + \underbrace{5y^4}_{d(y^5)})dy = 0$$

$$x^2y^2 + x^4 + y^5 = C$$

Consider $y' = xy^2 - \frac{2}{x}y - \frac{1}{x^3}$

a) verify that $f(x) = \frac{1}{x^2}$ satisfies the equation.

b) Solve the given equation.

a) $f' \stackrel{?}{=} xf^2 - \frac{2}{x}f - \frac{1}{x^3}$

$f(x) = \frac{1}{x^2} \qquad f' = \frac{-2}{x^3}$

$$xf^2 - \frac{2}{x}f - \frac{1}{x^3} = x\cdot\cancel{\frac{1}{x^4}} - \frac{2}{x}\cdot\frac{1}{x^2} - \cancel{\frac{1}{x^3}} = -\frac{2}{x^3}$$

so f satisfies the given eqn.

b) $y = \frac{1}{} + \frac{1}{} \Rightarrow y' = \frac{-2}{} - \frac{u'}{}$

$x^3 \qquad v^2$

Put in the eqn:

$$-\frac{2}{x^3} - \frac{v'}{v^2} = x\left(\frac{1}{x^2}+\frac{1}{v}\right)^2 - \frac{2}{x}\left(\frac{1}{x^2}+\frac{1}{v}\right) - \frac{1}{x^3}$$

$$-\frac{2}{x^3} - \frac{v'}{v^2} = x\left(\frac{1}{x^4}+\frac{1}{v^2}+\frac{2}{x^2 v}\right) - \frac{2}{x^3} - \frac{2}{xv} - \frac{1}{x^3}$$

$$\cancel{-\frac{2}{x^3}} - \frac{v'}{v^2} = \cancel{\frac{1}{x^3}} + \frac{x}{v^2} + \cancel{\frac{2}{xv}} - \cancel{\frac{2}{x^3}} - \cancel{\frac{2}{xv}} - \cancel{\frac{1}{x^3}}$$

$$- \qquad = \qquad v' = -x \Rightarrow v = -\frac{x^2}{2} + c$$

$$\int -x\,dx$$

$$y = \frac{1}{x^2} + \frac{1}{v} = \frac{1}{x^2} + \frac{1}{-\frac{x^2}{2}+c} \quad \square$$

$$f(x) = \frac{1}{x} \qquad y' = y^2 + \frac{2}{x}y - \frac{4}{x^2}$$

$$f' = f^2 + \frac{2}{x}f - \frac{4}{x^2}$$

$$f(x) = \frac{1}{x} \qquad f'(x) = -\frac{1}{x^2}$$

$$f^2 + \frac{2}{x}f - \frac{4}{x^2} = \frac{1}{x^2} + \frac{2}{x}\cdot\frac{1}{x} - \frac{4}{x^2} = -\frac{1}{x^2}$$

$$y = \frac{1}{x} + \frac{1}{v} \qquad y' = -\frac{1}{x^2} - \frac{v'}{v^2}$$

$$-\frac{1}{x^2} - \frac{v'}{v^2} = \left(\frac{1}{x}+\frac{1}{v}\right)^2 + \frac{2}{x}\left(\frac{1}{x}+\frac{1}{v}\right) - \frac{4}{x^2}$$

$$\cancel{-\frac{1}{x^2}} - \frac{v'}{v^2} = \cancel{\frac{1}{x^2}} + \frac{1}{v^2} + \frac{2}{xv} + \cancel{\frac{2}{x^2}} + \frac{2}{xv} - \cancel{\frac{4}{x^2}}$$

$$-\frac{v'}{v^2} = \frac{1}{v^2} + \frac{4}{xv} \qquad \text{(multiply by } -v^2\text{)}$$

$$v' = -1 - \frac{4}{x}v$$

$$v' + \frac{4}{x}v = -1 \longleftarrow \text{lin.}$$

$$\mu(x) = e^{\int \frac{4}{x}dx} = e^{4\ln x} = x^4$$

$$x^4 v' + 4x^3 v = -x^4$$

$$(x^4 v)' = -x^4 \Rightarrow x^4 v = -\int x^4 dx = -\frac{x^5}{5} + c$$

$$v = \frac{-\frac{x^5}{5} + c}{x^4}$$

$$y = \frac{1}{x} + \frac{1}{v} = \frac{1}{x} + \frac{x^4}{-\frac{x^5}{5} + c}$$